Capital-Skill Complementarity and Inequality Over the Business Cycle*

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Abstract

This study investigates the implications of capital-skill complementarity for the cyclical behavior of wage inequality. This is done in a dynamic general equilibrium model which extends the standard real business cycle model in three ways. First, the representative agent is replaced by two agent types, skilled and unskilled. Second, the standard, two-factor Cobb-Douglas production function is replaced by a more general, four-factor production function which allows for capital-skill complementarity. Third, the model includes both neutral and investment-specific technological change. The model successfully accounts for both the volatility and the cyclical behavior of the skill premium in the United States. The results of this study suggest that capital-skill complementarity may be an important determinant of wage inequality over the business cycle.

Keywords: business cycle, capital-skill complementarity, inequality, skill premium

JEL: E3; J31

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1 Introduction

Keane and Prasad (1993) found the aggregate skill premium in the United States to be uncorrelated with contemporaneous unemployment. This study confirms their finding and shows that the skill premium also lags the business cycle and exhibits a volatility which is roughly two-thirds that of output. The purpose of this study is to test whether or not these three facts regarding the cyclical behavior of the skill premium can be explained by the presence of capital-skill complementarity in production.

This is done in a dynamic general equilibrium model which extends the standard real business cycle model in three ways. First, the representative agent is replaced by two agent types, skilled and unskilled. Second, the standard, two-factor Cobb-Douglas production function is replaced by a more general, four-factor production function which allows for capital-skill complementarity. Third, the model includes both neutral and investment-specific technological change. The model is a synthesis of previous models by Greenwood et al. (1997) and Krusell et al. (2000).

The findings of this study suggest that capital-skill complementarity may be an important determinant of wage inequality over the business cycle. The model with capital-skill complementarity can account for both the volatility and the cyclical behavior of the skill premium in the United States. The model without capital-skill complementarity cannot.

In the model with capital-skill complementarity, the skill premium is driven by both movements in the ratio of unskilled to skilled hours worked and by the ratio of capital equipment to skilled hours worked. In both the data and the model simulations, the ratio of relative hours worked and the capital-skill ratio are uncorrelated with contemporaneous output. Together, they produce a skill premium which is uncorrelated with contemporaneous output. Furthermore, the ratio of relative hours worked leads the business cycle, while the capital-skill ratio lags the business cycle. The lag in the capital-skill ratio induces a lag in the skill premium, since the capital-skill complementarity effect dominates the relative supply effect.

The volatility of the skill premium in the model is determined by the volatility of investment-specific shocks together with the degree of capital-skill complementarity in production. Positive investment-specific shocks affect the skill premium by making new capital equipment more productive, which induces individuals to invest more in capital equipment as opposed to capital structures.[1] This increases the ratio of capital equipment to skilled labor which, in turn, raises the skill premium through the capital-skill complementarity mechanism.

Investment-specific shocks are not necessary, however, to match the other two stylized facts. The model with neutral technological shocks alone also produces a skill premium which is uncorrelated with contemporaneous output and lags the business cycle. In fact, these two qualitative results are extremely stable to both large parameter changes and to different specifications of the production function. Investment-specific shocks are important, however, for a sound quantitative analysis.

Until recently, it had been widely accepted that the skill premium in the United States moved countercyclically. This belief was based mainly on work done by Reder (1955, 1962) and was introduced into the dynamic macroeconomic literature by Kydland (1984). Since then, it has been recognized that estimates of the skill premium using aggregate data may be biased due to the changing composition of the workforce over the business cycle (Raisian, 1983; Keane and Prasad, 1993; Ziliak et al., 1999).[2] Using microeconomic panel data and controlling for this type of bias,

[1] Changes in the stock of capital structures do not affect the skill premium in this model, since the elasticity of substitution between capital structures and both unskilled and skilled labor is assumed to equal one.

[2] The aggregate skill premium may suffer from a procyclical bias, since employment variability is greatest among workers at the low end of the productivity distribution. Thus, the average quality of low skilled workers rises when unemployment is high and falls when unemployment is low, while the average productivity of high skilled workers remains roughly constant over the business cycle. This induces a procyclical movement in the relative productivities of these two groups and, subsequently, in the skill premium as well.

as well as correcting for potential selection bias, more recent empirical work has found the skill premium to be uncorrelated with contemporaneous measures of the business cycle (Keane and Prasad, 1993).[3]

All of these earlier studies rely upon different theories of implicit contracts in order to explain their empirical findings. There is never any mention of competitive wage setting. Keane and Prasad (1993), for example, discuss two types of theoretical models which can be used to help interpret their empirical findings concerning the cyclical behavior of hours and wages of workers with different skill levels. The implicit contract theories of Azariadis (1975, 1976), Bailey (1974), and Gordon (1974) support the existence of wage smoothing arrangements provided by firms for their workers. The implicit contract models of Hashimoto (1981) and Raisian (1983), on the other hand, imply more procyclical compensation for workers with higher skills. Both types of models rely on the existence of firm-specific capital (Becker, 1962) to create incentives to avoid costly separations of skilled workers from firms during recessions.

In stark contrast to these studies, this paper presents a simple, competitive theory of relative wages; one that succeeds in explaining the cyclical behavior of the skill premium. The success of the model presented in this paper hinges on the use of a production function characterized by capital-skill complementarity.

The effect of capital-skill complementarity on the relative demand for skilled labor has been addressed in a number of previous studies (for a review, see Hamermesh, 1993). One of the earliest and most influential pieces within this literature is Griliches's empirical test of the capital-skill complementarity hypothesis (Griliches, 1969).

More recently, The importance of capital-skill complementarity for explaining

[3] Ziliak et al. (1999) found that the returns to education (in the aggregate economy) for those with above average education levels were, in fact, weakly procyclical. But, their estimate was not significant and, hence, their study cannot reject the hypothesis that the aggregate skill premium is uncorrelated with contemporaneous output.

long run trends in wage inequality has been demonstrated by Krusell et al. (2000) and Lindquist (2000). Other researchers have illustrated the connection between capital-skill complementarity and wage dispersion (e.g. Caselli, 1999). This study makes an original contribution to this new literature by investigating the implications of capital-skill complementarity for the cyclical behavior of wage inequality. This study could also be viewed as a complement to the work of Castañeda et al. (1998) on income inequality over the business cycle.

The outline of the rest of the paper is as follows. In Section 2, the facts regarding the cyclical behavior of the skill premium are presented. An initial analysis of the importance of capital-skill complementarity is carried out in Section 3. This is done in two parts. First, a qualitative example is presented which demonstrates the potential impact of capital-skill complementarity on the cyclical behavior of the skill premium. Then, the empirical relevance of this hypothesis is tested. This test shows that the capital-skill ratio is, in fact, a significant determinant of the cyclical behavior of the skill premium. It motivates the extended analysis which follows.

In Section 4, a dynamic general equilibrium model is constructed which allows for capital-skill complementarity in production. This model is used to test whether or not the capital-skill complementarity mechanism can account both qualitatively and quantitatively for the cyclical behavior of the skill premium together with a number of standard business cycle facts. It can.

A sensitivity analysis is then carried out which demonstrates that the results which speak in favor of the model with capital-skill complementarity are robust to significant parameter changes. The sensitivity analysis also investigates how the skill premium behaves in a model without capital-skill complementarity and it highlights the role and importance of investment-specific technological shocks. The sensitivity analysis concludes with a brief comment on the potential bias which may exist in the aggregate data on the skill premium used in this study.

Table 1: The Cyclical Behavior of the Aggregate U.S. Skill Premium

	volatility[a]	cross-correlations of x(t) and skill premium (sp)					
skill premium[b]	1.90	sp(t-1)	sp(t)	sp(t+1)	sp(t+2)	sp(t+3)	sp(t+4)
output[b]	3.02	-.01	-.20	-.22	-.06	0.09	0.16
unemployment[c]	1.51	0.02	0.10	0.05	-.11	-.20	-.21

a) Volatility is measured as the standard deviation of percentual fluctuations around trend.

b) Annual U.S. data, 1963-1992, from Krusell et al. (2000).

c) Annual U.S. data, 1963-1992, from the U.S. Department of Labor, Bureau of Labor Statistics' *Current Population Survey.*

A final test, comparing the impulse response of the U.S. skill premium to output shocks with the impulse response of the model skill premium to output shocks, is carried out in Section 5. This test also speaks in favor of the model with capital-skill complementarity and its ability to explain movements in the skill premium over the business cycle. Section 6 concludes.

2 The Cyclical Behavior of the Skill Premium

Using output and unemployment as measures of the business cycle, we can begin examining the cyclical behavior of the skill premium by first looking at a set of simple correlations.[4] These are shown in Table 1 along with the volatilities of output, unemployment and the skill premium. The contemporaneous correlation of output and the skill premium is -0.20. The skill premium lags output, peaking at $t+4$ with a correlation coefficient of 0.16. The contemporaneous correlation of unemployment and the skill premium is 0.10. The skill premium lags this measure of the business cycle as well, peaking at $t+4$ with a correlation coefficient of -0.21.[5]

Table 1 is clear on two facts regarding the cyclical behavior of the skill premium;

[4] The data used in this study (unless otherwise specified) is taken from Krusell et al. (2000). Their data set consists of aggregate annual U.S. time series between 1963 and 1992, including; output, capital structures, capital equipment, the relative price of capital equipment, skilled and unskilled labor inputs, and the skill premium. They define skilled workers as those workers with 16 or more years of schooling. Their time series are detrended using an H-P filter with $\lambda = 100$ for use in this study.

Figures on annual unemployment, 1963-1992, are from the U.S. Bureau of Labor Statistics. This time series is also detrended using an H-P filter with $\lambda = 100$.

[5] Similar correlations are obtained using first differences.

the skill premium lags the business cycle and its volatility is roughly two-thirds that of output. It is less clear on whether the contemporaneous correlation between the skill premium and the business cycle is weakly negative or essentially zero.

Alternatively, we can examine the cyclical behavior of the skill premium by constructing a VAR(p) model of output, y_t, the inverse of unemployment, $1/un_t$, and the skill premium, $w_{s,t}/w_{u,t}$. A VAR(1) model without a constant is chosen as the best representation of the model, since it minimizes both the Akaike information criteria and the Schwarz criteria. The VAR(1) model can be written as

$$\begin{bmatrix} y_t \\ \frac{1}{un_t} \\ \frac{w_{s,t}}{w_{u,t}} \end{bmatrix} = \mathbf{\Delta} \begin{bmatrix} y_{t-1} \\ \frac{1}{un_{t-1}} \\ \frac{w_{s,t-1}}{w_{u,t-1}} \end{bmatrix} + \boldsymbol{\varepsilon}_t \tag{1}$$

$$\boldsymbol{\varepsilon}_t \sim N\left(0, \mathbf{\Sigma}\right)$$

where $\mathbf{\Delta}$ is a 3×3 matrix of OLS regression coefficients and where $\boldsymbol{\varepsilon}_t$ is a 3×1 vector of normally distributed $i.i.d.$ shocks with mean zero and variance-covariance matrix $\mathbf{\Sigma}$.

The impulse response functions of the skill premium to a one standard deviation shock to output and to a one standard deviation shock to the inverse of unemployment can be seen in Figure 1. These impulse response functions show us two important facts about the cyclical behavior of the skill premium. First, they show us that the skill premium is essentially uncorrelated with both contemporaneous output and the inverse of contemporaneous unemployment, which confirms the findings of Keane and Prasad (1993). Second, they shows us that the skill premium lags the business cycle, which confirms what we saw in Table 1. This paper will attempt to account for these two facts and for the observed level of volatility in the skill premium using a dynamic general equilibrium model with a production function which is characterized by capital-skill complementarity.

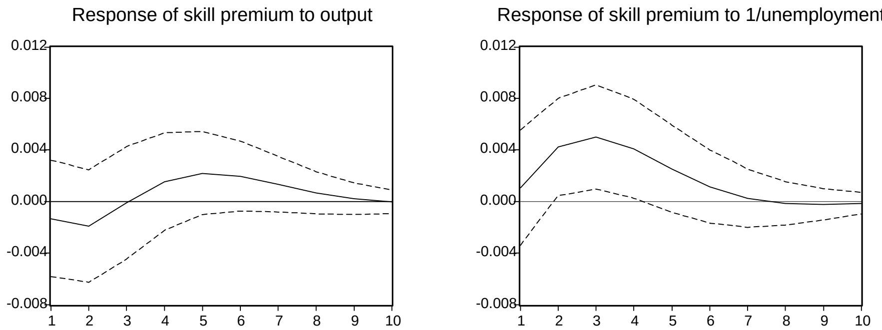

Figure 1: Impulse Response Functions of the Skill Premium to a One S.D. Shock to Output and to a One S.D. Shock to the Inverse of Unemployment (along with ± 2 s.d. bands).

3 An Initial Analysis of the Importance of Capital-Skill Complementarity

The potential effect of capital-skill complementarity upon the cyclical behavior of the skill premium can be demonstrated with a simple, qualitative example borrowed from Krusell et al. (2000). First, assume that capital, k_t, and skilled hours worked, $h_{s,t}$, are complements, while capital and unskilled hours worked, $h_{u,t}$, are substitutes. For ease of exposition, assume also that capital and unskilled hours are perfect substitutes and that they have an elasticity of substitution with skilled hours equal to one. Given these assumptions, the production function of a firm can be written as

$$f\left(k_t, h_{s,t}, h_{u,t}\right) = \left(k_t + h_{u,t}\right)^{\theta} h_{s,t}^{1-\theta}. \tag{2}$$

The skill premium can be derived from this function using the first order, profit maximizing conditions of the firm under perfect competition

$$\frac{f_{h_{s,t}}}{f_{h_{u,t}}} = \frac{w_{s,t}}{w_{u,t}} = \left(\frac{1-\theta}{\theta}\right) \frac{k_t + h_{u,t}}{h_{s,t}}. \tag{3}$$

Taking the derivatives of the skill premium with respect to relative hours worked and to the capital-skill ratio gives us the following partial derivatives (i.e. theoretical predictions)

$$\frac{\partial \frac{w_{s,t}}{w_{u,t}}}{\partial \frac{h_{u,t}}{h_{s,t}}} > 0, \; \frac{\partial \frac{w_{s,t}}{w_{u,t}}}{\partial \frac{k_t}{h_{s,t}}} > 0.$$

Thus, when production is characterized by capital-skill complementarity, an increase in the ratio of capital to skilled hours worked raises *ceteris paribus* the skill premium. An increase in the ratio of unskilled to skilled hours worked also raises *ceteris paribus* the skill premium. In contrast to this, the skill premium associated with a Cobb-Douglas production function is a function of relative hours only.[6]

To test the empirical relevance of these theoretical predictions, let us examine a model of the skill premium with capital-skill complementarity using the Krusell et al. (2000) data. The model includes output, y_t, relative hours worked, $h_{u,t}/h_{s,t}$, the capital-skill ratio, $k_t/h_{s,t}$ and the skill premium, $w_{s,t}/w_{u,t}$. The preferred model specification is a VAR(1) without a constant[7]

$$\begin{bmatrix} y_t \\ \frac{h_{u,t}}{h_{s,t}} \\ \frac{k_t}{h_{s,t}} \\ \frac{w_{s,t}}{w_{u,t}} \end{bmatrix} = \mathbf{\Delta} \begin{bmatrix} y_{t-1} \\ \frac{h_{u,t-1}}{h_{s,t-1}} \\ \frac{k_{t-1}}{h_{s,t-1}} \\ \frac{w_{s,t-1}}{w_{u,t-1}} \end{bmatrix} + \boldsymbol{\varepsilon}_t \tag{4}$$

$$\boldsymbol{\varepsilon}_t \sim N(0, \mathbf{\Sigma})$$

where $\mathbf{\Delta}$ is a 4×4 vector of OLS regression coefficients and where $\boldsymbol{\varepsilon}_t$ is a 4×1 vector of normally distributed *i.i.d.* shocks with mean zero and variance-covariance matrix $\mathbf{\Sigma}$.

Examining the impulse response function of the skill premium to a one stan-

[6] If the production function is Cobb-Douglas, then $f_{h_{s,t}}/f_{h_{u,t}} = w_{s,t}/w_{u,t} = \theta_s h_{u,t}/\theta_{u,t} h_{s,t}$, where θ_s and θ_u are the income shares of skilled and unskilled labor, respectively.

[7] This representation of the model is chosen since it minimizes both the Akaike information criteria and the Schwarz criteria.

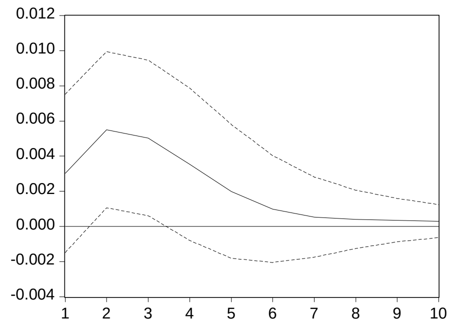

Figure 2: Impulse Response of the Skill Premium to a One S.D. Shock to the Capital-Skill Ratio (along with $\pm$ 2 s.d. bands).

dard deviation shock to the capital-skill ratio (see Figure 2), we find that it has a significant, persistent and positive impact on the skill premium, just as the theory predicts. We also see that the impact of the capital-skill ratio on the skill premium is strongest in the second and third periods after the initial shock. These findings can be confirmed by running a simple OLS regression on the U.S. data, which results in the following regression equation

$$\frac{w_{s,t}}{w_{u,t}} = \underset{(0.0006)}{0.0012}\frac{k_{t-1}}{h_{s,t-1}} + \underset{(0.0137)}{0.0345}h_{u,t} - \underset{(0.0238)}{0.0702}h_{s,t} \tag{5}$$

with an $\overline{R}^2 = 0.41$ and DW $= 1.56$. Once again, we find the ratio of capital to skilled labor plays a significant role in determining the cyclical behavior of the skill premium, albeit with a one period lag.[8] The empirical finding that the (detrended) capital-skill ratio has a significant impact on the (detrended) skill premium speaks in favor of a model in which production is characterized by capital-skill complementarity and against the use of a model with a standard, Cobb-Douglas production function.

[8]Using output, lagged output, unemployment or lagged unemployment as control variables has little or no impact on the size or significance of the coefficient of the capital-skill variable.

4 An Extended Analysis

In this section, a dynamic general equilibrium model is constructed which allows for capital-skill complementarity in production. This model will be used to test whether or not the capital-skill complementarity mechanism can account both qualitatively and quantitatively for the cyclical behavior of the skill premium together with a number of standard business cycle facts.

4.1 The Model

Consider an economy inhabited by two types of infinitely lived agents; skilled, s, and unskilled, u, where $u + s = 1$. Agents are born at time zero and acquire their respective skill (or human capital) endowments at birth. Agents have CRRA utility functions

$$U_{i,t}\left(c_{i,t}, l_{i,t}\right) = \frac{\left(c_{i,t}^{\alpha} l_{i,t}^{1-\alpha}\right)^{1-\gamma} - 1}{1-\gamma} \tag{6}$$

$$\alpha < 0 < 1, \;\; \gamma \geq 0.$$

They derive utility from consumption, $c_{i,t}$, and leisure, $l_{i,t}$, where $i \in \{u, s\}$ and t is a time subscript. Each agent is endowed with one unit of labor, so that hours worked by an agent, $h_{i,t}$, equal

$$h_{i,t} = 1 - l_{i,t}. \tag{7}$$

There is a continuum of firms in the economy which have identical production functions. Firms use capital structures, capital equipment, skilled labor and unskilled labor to produce output and they behave competitively in product and factor markets.

Assuming constant returns to scale in production allows us to aggregate across firms without loss of generality. Let $K_{s,t}$ and $K_{e,t}$ denote the aggregate stock of structures and equipment, respectively, and let $H_{i,t} = ih_{i,t}$ denote aggregate hours worked by type i agents. Aggregate production, Y_t, is then given by[9]

$$Y_t = e^{(z_t)} K_{s,t}^{\theta} \left[\mu H_{u,t}^{\nu} + (1-\mu) \left[\lambda K_t^{\varphi} + (1-\lambda) H_{s,t}^{\varphi} \right]^{\frac{\nu}{\varphi}} \right]^{(1-\theta)/\nu} \tag{8}$$

$$\theta,\ \mu,\ \lambda \in (0,1)\,;\ \nu,\ \varphi \in (-\infty, 1)\,;\ \nu,\ \varphi \neq 0$$

where z_t is a random productivity parameter which follows an AR(1) process given by

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t} \tag{9}$$

$$\varepsilon_z \sim N\left(\mu_z, \sigma_z^2\right),\ E\left[\mu_z\right] = 0$$

where ρ_z is the persistence parameter and ε_z are drawn from a bivariate normal distribution and have mean zero and variance σ_z^2 (more on this below).

The parameters ν and φ are the two key substitution parameters. The income share of capital structures is given by θ, while μ and λ are the income share parameters of unskilled and skilled agents, respectively. The elasticity of substitution between capital equipment and skilled labor is equal to $1/(1-\varphi)$. The elasticity of substitution between capital equipment and unskilled labor and the elasticity of substitution between skilled and unskilled labor are both equal to $1/(1-\nu)$. If $\nu > \varphi$, then the production function is said to exhibit capital-skill complementarity.

The skill premium associated with the above production function is

$$\frac{w_{s,t}}{w_{u,t}} = \frac{(1-\mu)(1-\lambda)}{\mu} \left[\lambda \left(\frac{K_{e,t}}{H_{s,t}} \right)^{\varphi} + (1-\lambda) \right]^{\frac{\nu-\varphi}{\varphi}} \left(\frac{H_{u,t}}{H_{s,t}} \right)^{1-\nu}. \tag{10}$$

Examining this equation, we see that when $\nu > \varphi$, a rise in the stock of capital equipment will *ceteris paribus* raise the skill premium. Krusell et al. (2000) call

[9] The production function used in this paper is similar to the one estimated in Krusell et al. (2000).

this the *capital-skill complementarity effect.* We also see that a rise in the ratio of unskilled to skilled hours worked will *ceteris paribus* raise the skill premium for any acceptable values of ν and φ. Krusell et al. (2000) call this the *relative supply effect.* Equation 10 shows us that the cyclical behavior of the skill premium in the model with capital-skill complementarity is dependent upon both the cyclical behavior of the ratio of unskilled to skilled hours worked and upon the cyclical behavior of the ratio of capital equipment to skilled labor.

The law of motion for aggregate capital structures is

$$K_{s,t+1} = (1-\delta_s)K_{s,t} + X_{s,t} \tag{11}$$

$$0 \leq \delta_s \leq 1$$

where $X_{s,t}$ is aggregate investment in new capital structures and δ_s is the depreciation rate of capital structures. The law of motion for aggregate capital equipment is

$$K_{e,t+1} = (1-\delta_e)K_{e,t} + X_{e,t}e^{(q_t)} \tag{12}$$

$$0 \leq \delta_e \leq 1$$

where $X_{e,t}$ is aggregate investment in new capital equipment and δ_e is the depreciation rate of capital equipment.

The accumulation equation for capital equipment differs from that of capital structures, since it includes a factor, q_t, which represents the current state of technology for producing equipment. It determines the amount of equipment that can be purchased for one unit of output. Changes in q_t formalize the notion of investment-specific technological change, while changes in z_t represent neutral technological change.

The inclusion of investment-specific technological change in the model is motivated by the observation that technological change specific to equipment has been far more dramatic over the past three decades than for structures.[10] Previous work has shown it to be an important part of an explanation of rising wage inequality between skilled and unskilled workers in the U.S. (Krusell et al., 2000). Greenwood et al. (1997) show that investment-specific technological can account for 58 percent of output growth in the United States. In a related paper, Greenwood et al. (2000) argue that investment-specific technological change is the source of about 30 percent of fluctuations in output over the business cycle. In this paper, we shall see that investment-specific technological change plays an important role in determining the cyclical behavior of the skill premium. It affects the skill premium through the capital-skill complementarity mechanism.

In contrast to Greenwood et al. (1997), shocks to investment-specific technology and shocks to neutral technology will be allowed to covary. The correlation coefficient between the two types of shocks, ρ_{zq}, is given by $|\sigma_{yz}/\sigma_y\sigma_z| \leq 1$. The stochastic processes governing the movement of q_t is modeled as an AR(1) process

$$q_t = \rho_q q_{t-1} + \varepsilon_{q,t} \tag{13}$$

$$(\varepsilon_q \mid \varepsilon_z) \sim N\left[\left(\mu_q - \eta\mu_z\right) + \eta\varepsilon_z, \varpi^2\right]$$

$$\eta = \frac{\sigma_{qz}}{\sigma_z^2},\ \varpi^2 = \sigma_q^2 - \frac{\sigma_{qz}^2}{\sigma_z^2},\ E\left[\mu_q\right] = 0$$

where ρ_q is the persistence parameter and ε_q are drawn from a bivariate normal

[10] This view is motivated in Greenwood et al. (1997), Greenwood et al. (2000) and in Krusell et al. (2000) by the following empirical observations. First, that the relative price of structures appears to be stationary in the U.S. data, as does the ratio of structures to output. Second, that the relative price of capital equipment has fallen dramatically since 1974, while the ratio of equipment to output has risen. This view is supported by the empirical work of Gordon (1989, 1990) which describes the rise in productivity in the IT producing sector over the last three decades, while Pieper (1989) describes a fall in the productivity of the construction sector during this period (although he argues that this fall may be overexaggerated due to the use of poor statistical methods in accounting for structures).

distribution[11] and have mean $\left(\mu_q - \eta\mu_z\right) + \eta\varepsilon_z$ and variance ϖ^2. The unconditional mean of shocks to investment-specific technology, μ_q, is zero.

4.1.1 The Social Planner's Problem

If we imagine that this economy is governed by a benevolent social planner, the problem faced by the planner is to choose sequences for consumption, labor supply, and capital, that, given $K_{e,0}$, $K_{s,0}$, s, and u, maximizes the weighted sum of the expected utilities of the two groups of agents

$$\max_{\{c_{s,t},c_{u,t},h_{u,t},h_{s,t},K_{s,t+1},K_{e,t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\Psi)\, uU_{u,t}\left(c_{u,t}, l_{u,t}\right) + \Psi sU_{s,t}\left(c_{s,t}, l_{s,t}\right) \right\} \tag{14}$$

subject to the aggregate resource constraint

$$C_{s,t} + C_{u,t} + X_{s,t} + X_{e,t} = Y_t \tag{15}$$

and to Equations 6 - 9 and 11 - 13, where $C_{i,t} = ic_{i,t}$, β is the discount factor of the social planner and Ψ is the social planner's weight on skilled utility.

4.1.2 Model Equilibrium and Solution

An equilibrium for this economy consists of a set of decision rules $h_i(K_e, K_s, z, q)$ and $c_i(K_e, K_s, z, q)$ for $i \in \{u, s\}$, such that (i) the decision rules solve the social planner's welfare maximization problem and (ii) prices are equal to marginal products.

The decision rules $h_i\left(K_e, K_s, z, q\right)$ and $c_i\left(K_e, K_s, z, q\right)$ can be found in the following manner. First, the Lagrangian function, $\mathcal{L}$, associated with the social planner's problem is constructed

$$\mathcal{L}\left(K_{e,t}, K_{s,t}, c_{i,t}, h_{i,t}, \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}\right) = \tag{16}$$

[11] See appendix A2.8 in Hendry (1995) for the derivation of the marginal distribution of ε_{z_t} and the conditional distribution of ε_{q_t}.

$$
\begin{aligned}
&E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1-\Psi)\, u \frac{\left[c_{u,t}^{\alpha} (1-h_{u,t})^{1-\alpha}\right]^{1-\gamma} - 1}{1-\gamma} + \Psi s \frac{\left[c_{s,t}^{\alpha} (1-h_{s,t})^{1-\alpha}\right]^{1-\gamma} - 1}{1-\gamma} \right\} \\
&-E_0 \sum_{t=0}^{\infty} \beta^t \lambda_{1,t} [C_{s,t} + C_{u,t} + K_{s,t+1} - (1-\delta_s) K_{s,t} + ... \\
&K_{e,t+1}/e^{(q_t)} - (1-\delta_e) K_{e,t}/e^{(q_t)} - e^{(z_t)} F\left(K_{e,t}, K_{s,t}, H_{s,t}, H_{u,t}\right)] \\
&-E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \lambda_{2,t} \left[z_t - \rho_z z_{t-1} - \varepsilon_{z,t} \right] \right\} \\
&-E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \lambda_{3,t} \left[q_t - \rho_q q_{t-1} - \varepsilon_{q,t} \right] \right\}
\end{aligned}
$$

where $\lambda_{1,t}$, $\lambda_{2,t}$ and $\lambda_{3,t}$ are the Lagrange multipliers associated with the aggregate resource constraint, the law of motion for z_t and the law of motion for q_t, respectively and where $e^{(z_t)} F(.)$ is aggregate production. The agents' time constraints have been substituted directly into their utility functions and the laws of motion for aggregate capital structures and equipment have been substituted into the aggregate resource constraint.

From this problem we can derive the necessary first-order conditions for an optimal solution to the social planner's problem. For convenience sake, $\lambda_{1,t}$ is eliminated from the set of equations. We are then left with a system of eight equations in eight unknowns which characterizes the equilibrium processes of the model.

The next step is to solve this system for its steady state values. The entire system of equations can then be log linearized around these steady state values. The resulting system of linear expectational difference equations is solved using the generalized Schur method as described in Klein (2000). The solution to this problem delivers the decision rules, $h_i(K_e, K_s, z, q)$ and $c_i(K_e, K_s, z, q)$.

4.2 Model Calibration

The model is calibrated as follows. First, the proportion of skilled workers in the economy, s, is set equal to 24 percent, which is equal to the percentage of the U.S.

labor force in 1992 which had 16 or more years of schooling (OECD, 1995). This definition of skills is consistent with that used by Krusell et al. (2000).

The key substitution parameters, $\nu = 0.401$ and $\varphi = -0.495$, and the income share of capital structures, $\theta = 0.117$, were estimated in Krusell et al. (2000). The depreciation rate of capital structures, $\delta_s = 0.056$, and capital equipment, $\delta_e = 0.124$, are taken from Greenwood et al. (1997). Agents are assumed to be risk averse with γ equal to 2 and the discount rate, β, is set equal to 0.95.

The income share parameters, $\mu = 0.411$ and $\lambda = 0.475$, the social planner's weight on skilled utility, $\Psi = 0.487$, and the consumption share in utility, $\alpha = 0.576$, are calibrated simultaneously. They are calibrated so that in the steady state agents work (on average) 40 hours a week, skilled workers work 19 percent more than unskilled workers (Welch, 1997), the skilled wage is 73 percent higher than the unskilled wage (OECD, 1995), and the income share of capital is 30 percent, which is the same as that found in the Krusell et al. (2000) data.

The AR(1) process governing investment specific technological shocks is calibrated following Greenwood et al. (2000), with ρ_q equal to 0.64 and σ_z equal to 0.035. The persistence parameter of the neutral technological process, ρ_z, is set equal to 0.815, while σ_z is scaled so that the volatility of output in the model is always the same as that found in the Krusell et al. (2000) data. In the benchmark case, σ_z is equal to 0.0265.

Examining the detrended[12] time series for investment-specific technological change, q, and neutral technological change, z, (see Figure 3), there appears to be a negative correlation between the two time series. This correlation is equal to -0.35.

[12] The time series for q and z are estimated and detrended in a manner similar to that used by Greenwood et al. (2000). Investment-specific technological change is estimated as follows. First, let Q equal the inverse of the relative price of capital equipment, which is taken from the Krusell et al. (2000) data set. Then, estimate $\ln Q_t = \text{constant} + \text{trend} + q_t$, where $q_t = \rho_q q_{t-1} + \varepsilon_{q,t}$. Neutral technologcial change is estimated as follows. First, let $Z_t = Y_t / F\left(K_{e,t}, K_{s,t}, H_{s,t}, H_{u,t}\right)$, where Y_t, $K_{e,t}$, $K_{s,t}$, $H_{s,t}$ and $H_{u,t}$ are taken from the Krusell et al. (2000) data set. Then, estimate $\ln Z_t = \text{constant} + \text{trend} + z_t$, where $z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$.

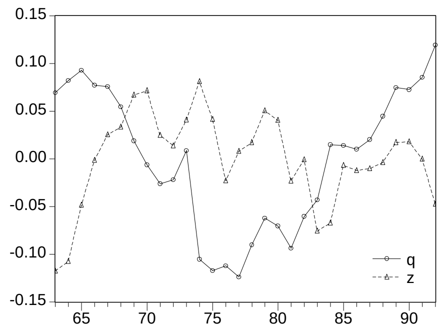

Figure 3: Detrended Investment-Specific Technological Change, q, and Detrended Neutral Technological Change, z.

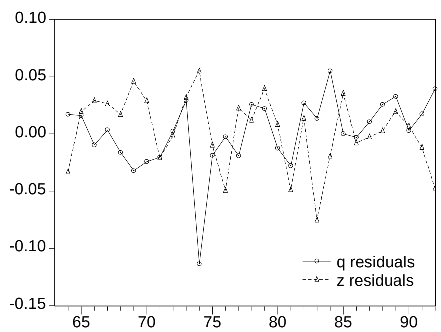

Figure 4: Shocks to Investment-Specific Technological Change, q Residuals, and Shocks to Neutral Technological Chnage, z Residuals.

More importantly, their residuals (see Figure 4) are also negatively correlated with a correlation coefficient, ρ_{zq} = -0.31.

4.3 Business Cycle Properties of the Model

In this section, the general business cycle behavior of the model is discussed and evaluated. The main conclusions are that the model performs well along many, but not all, of the standard business cycle measures and that it performs at least as well as a standard, homogeneous-agent business cycle model. Furthermore, the model with capital-skill complementarity matches the data concerning movements in the skill premium over the business cycle, while the model without capital-skill complementarity does not.

Business cycle statistics from the U.S. National Income and Product Accounts (NIPA), from the Krusell et al. (2000) data set, and from the model are summarized in Tables 2, 3 and 4. All statistics, for the U.S. data and the model economy, are calculated on detrended, annual variables. The variables were detrended using an H-P filter with $\lambda = 100$. The NIPA data is included as a familiar standard, but does not include per capita or relative variables concerning skilled and unskilled workers. Data from Krusell et al. (2000) are used to calculate these statistics.

The business cycle properties of the aggregate real variables in the U.S. economy are well known. All of them are highly procyclical, except the capital stock which is uncorrelated with contemporaneous output. Investment is much more volatile than output and hours worked, which are, in turn, more volatile than consumption and the capital stock. Productivity leads the business cycle and the capital stock lags the business cycle. All other real variables peak at the same time as output.

In the model, all aggregate variables are highly procyclical with the exception of the capital stock which is uncorrelated with contemporaneous output and lags the business cycle. This lag (which we also observe in the data) will play an important role for explaining movements of the skill premium over the business cycle in the model. The model does a particularly good job of matching the volatility and cyclical behavior of output, capital, investment and hours worked. The volatility of consumption (relative to that of output), on the other hand, is lower than that in the data and the measure of productivity is strongly procyclical. Overall, the model does as well as any standard real business cycle model at matching the cyclical behavior of the aggregate U.S. economy (see e.g. Cooley and Prescott, 1995).

The dynamic behavior of the aggregate variables in this model can also be illustrated using the impulse response functions generated by the model. If we first

Table 2: Volatility[a] of Variables

Variable	NIPA data[b] 1963-1992	Krusell et al. data[c] 1963-1992	Model data
output	2.13	3.02	3.02 (0.27)[d]
consumption	1.85	n.a.	1.87 (0.18)
investment	7.91	n.a.	10.70 (1.22)
hours	2.09	1.11	1.11 (0.12)
productivity	0.93	2.95	2.21 (0.19)
capital	0.67	1.08	1.47 (0.23)
skill premium	n.a.	1.90	1.04 (0.15)
equipment/skilled hours	n.a.	2.50	4.50 (0.50)
unskilled/skilled hours	n.a.	2.15	1.00 (0.09)
skilled wage	n.a.	2.07	2.35 (0.20)
unskilled wage	n.a.	2.78	2.24 (0.20)
skilled hours	n.a.	2.08	1.29 (0.15)
unskilled hours	n.a.	1.22	1.16 (0.11)

a) Volatility is measured as the standard deviation of percentual fluctuations around trend.
b) Annual data from the U.S. National Income and Product Accounts.
c) Annual data from Krusell et al. (2000).
d) Standard deviations of the sample distributions (from the simulations) are in parentheses.

Table 3: Correlations of Aggregate Variables with Output

Variable x	Cross-Correlations of output (t) with					
	x(t-1)		x(t)		x(t+1)	
	NIPA[a] 1963-1992	Model	NIPA 1963-1992	Model	NIPA 1963-1992	Model
output	0.49	0.42 (0.09)[b]	1.00	1.00 (0.00)	0.49	0.42 (0.09)
consumption	0.71	0.20 (0.10)	0.90	0.81 (0.05)	0.34	0.48 (0.09)
investment	0.48	0.48 (0.09)	0.86	0.90 (0.03)	0.15	0.27 (0.10)
hours	0.24	0.49 (0.09)	0.91	0.81 (0.04)	0.68	0.19 (0.10)
productivity	0.64	0.33 (0.09)	0.29	0.95 (0.01)	-.43	0.48 (0.09)
capital	-.41	-.31 (0.08)	0.07	0.01 (0.07)	0.80	0.60 (0.02)

a) Annual data from the U.S. National Income and Product Accounts.
b) Standard deviations of the sample distributions (from the simulations) are in parentheses.

examine the impulse response of the aggregate variables to a positive shock to neutral technological change (see Figure 5), we see that all of the aggregate variable have an immediate and positive response, with the exception of the capital stock which shows a pronounced lag.

Unlike neutral technological shocks, which enter the production function directly, investment-specific technological shocks can only affect current output by encouraging labor to work more or less in the current period. They do, however, have a direct impact on the quality of current investments in capital equipment. As we can see in Figure 6, a positive shock to investment-specific technology raises investment and lowers consumption in the current period. Hours worked also increase, which helps to finance increased investment and allows for a rise in the marginal product of capital, despite the fact that capital in the current period is fixed and that the shock does not enter the production function directly. Thus, both output and hours respond immediately to investment-specific shocks, but they do not peak until the period following the shock.

The total capital stock increases in response to investment-specific shocks, albeit with a strong lag. These shocks also affect the composition of the capital stock. The amount of capital equipment in the economy is now greater than it was before the shock (in both absolute terms and relative to structures). It is now more profitable for agents to invest in capital equipment and less in structures.[13] This portfolio shift affects relative wages in the model since capital equipment and skilled labor are assumed to be complementary.

Now, let us turn our attention to the business cycle statistics concerning the per capita and relative variables. In particular, we want to examine the cyclical properties of the skill premium. But, we also want to look at the behavior of the

[13] Investments in new structures are still positive. But, they are not large enough to offset depreciation. This is why we observe a negative impulse response of capital structures to investment-specific technology shocks. It is not the case that investors are reversing previous investment decisions by converting old structures into new equipment.

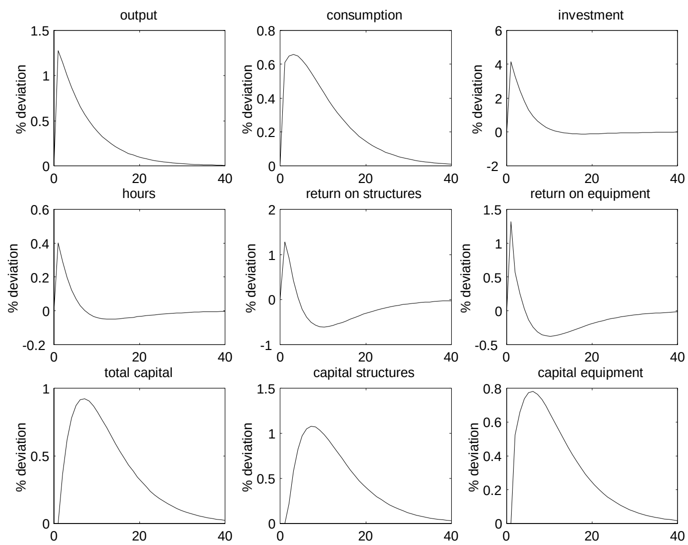

Figure 5: Impulse Response of Aggregate Variables to Neutral Productivity Shock.

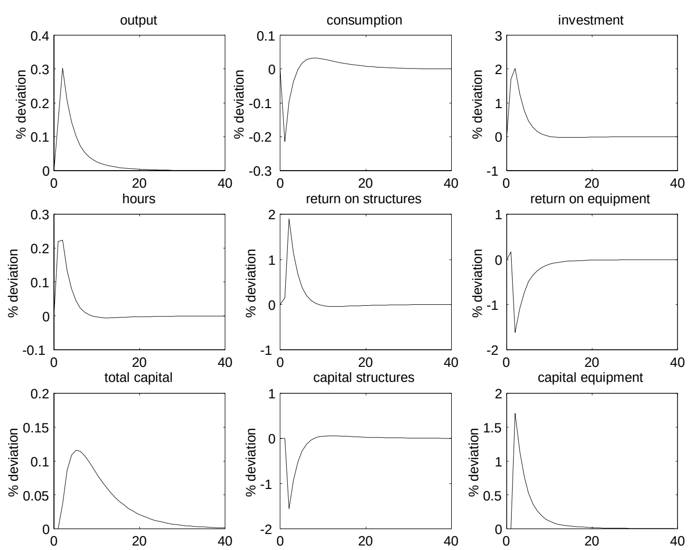

Figure 6: Impulse Response of Aggregate Variables to Investment-Specific Productivity Shock.

Table 4: Correlations of Per Capita and Relative Variables with Output

Variable x	Cross-Correlations of output (t) with					
	x(t-1)		x(t)		x(t+1)	
	Krusell[a]	Model	Krusell	Model	Krusell	Model
skill premium	-.01	-.07	-.20	0.10	-.22	0.12
		(0.13)[b]		(0.12)		(0.13)
equipment/skilled	-.30	-.13	0.18	0.00	0.19	0.11
		(0.13)		(0.12)		(0.13)
unskilled/skilled	0.13	0.23	0.05	0.15	-.07	-.07
		(0.12)		(0.11)		(0.12)
skilled wage	-.27	0.29	-.16	0.94	-.11	0.49
		(0.09)		(0.01)		(0.09)
unskilled wage	-.32	0.34	-.02	0.94	0.10	0.45
		(0.09)		(0.02)		(0.09)
skilled hours	0.40	0.29	0.12	0.61	0.04	0.20
		(0.12)		(0.09)		(0.13)
unskilled hours	0.77	0.52	0.24	0.82	-.26	0.17
		(0.09)		(0.03)		(0.09)

a) Annual data, 1963-1992, from Krusell et al. (2000).

b) Standard deviations of the sample distributions (from the simulations) are in parentheses.

ratio of equipment to skilled hours worked and of the ratio of relative hours worked, since it is these two factors which determine the skill premium in the model.

In Table 2, we see that the volatility of the model skill premium is equal to 55 percent of that found in the data. The volatility of the equipment-skill ratio in the model is 80 percent higher than that found in the data, while the ratio of relative hours worked is about half that found in the data.

Although not as volatile as in the data, we see in Table 4, that the skill premium in the model matches the other two stylized facts concerning the cyclical behavior of the skill premium presented in Section 2. It is essentially uncorrelated with contemporaneous output and it lags the business cycle. The skill premium in the model has a correlation coefficient with contemporaneous output equal to 0.10 (0.12).[14] The

[14]The numbers in parentheses are standard deviations of the sample distributions from the simulations.

correlation in the data is -0.20. The skill premium in the model lags the business cycle, peaking at time $t+2$ with a correlation coefficient with time t output of 0.17 (0.13) (see Table 5). The skill premium in the U.S. data also lags the business cycle, peaking at time $t+4$ with a correlation coefficient with time t output of 0.16 (see Table 5).

Table 4 also shows us that this model does a very good job of tracking the cyclical behavior of the equipment to skill ratio and of the ratio of relative hours worked. This is important, since it is these two variables that drive the skill premium in the model. Both are essentially uncorrelated with contemporaneous output. Together, they produce a skill premium which is also uncorrelated with contemporaneous output. In both the data and the model simulations, the ratio of relative hours leads the cycle, while the capital-skill ratio lags the business cycle. This lag induces a lag in the skill premium, since the capital-skill complementarity effect dominates the relative supply effect.[15]

The dynamic behavior of the per capita and relative variables can also be understood by examining the impulse response functions generated by the model. In Figure 7, we see that a positive shock to neutral technological change raises the wages of both skilled and unskilled workers. Higher wages induce both types of labor to work more hours. The increase in the supply of unskilled hours, however, is stronger than that of skilled hours, since unskilled workers have a lower marginal disutility of labor. Thus, the ratio of unskilled to skilled hours worked initially rises in response to a neutral technology shock.

The ratio of capital equipment to skilled hours worked falls initially, since skilled hours respond more quickly than the stock of capital equipment to productivity shocks. Thus, the relative supply effect and the capital-skill complementarity effect

[15] The model does a rather poor job, however, of tracking the Krusell et al. (2000) data concerning hours worked by skilled and unskilled workers and their wages (see table 4). The model variables are too strongly procyclical (a common result in the RBC paradigm). The model does a much better job, however, at tracking total hours worked as reported in the NIPA data (see Table 2).

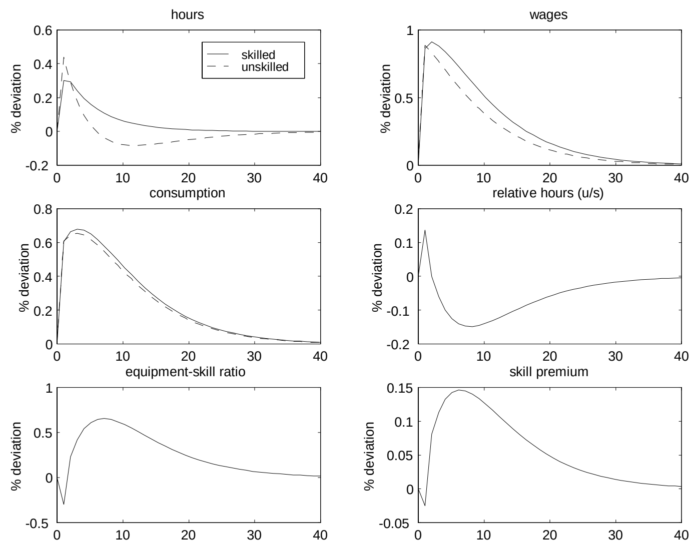

Figure 7: Impulse Response of Per Capita and Relative Variables to Neutral Productivity Shock.

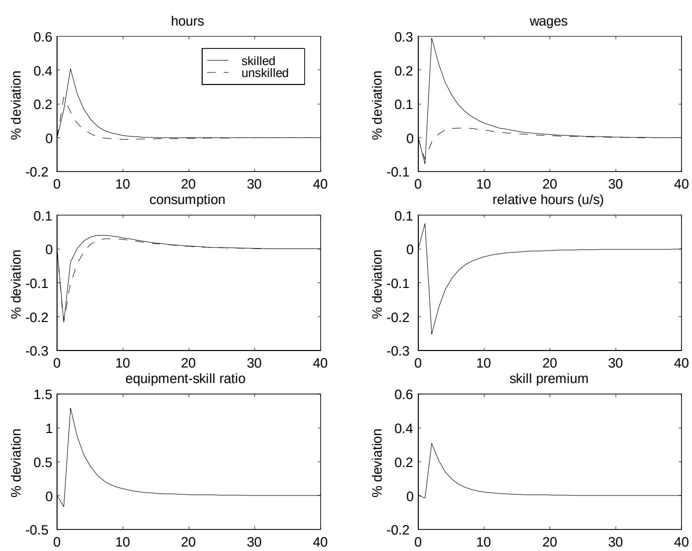

Figure 8: Impulse Response of Per Capita and Relative Variables to Investment-Specific Productivity Shock.

move in opposite directions in response to neutral productivity shocks, resulting in a skill premium which is essentially uncorrelated with contemporaneous shocks.

In Figure 8, we see that wages and consumption initially fall in response to a positive shock to investment-specific technology. Falling wages are due to an increase in the supply of both types of labor. For unlike neutral technology shocks, which enter the production function directly, investment-specific shocks cannot support wages in the face of an increasing supply of labor. But, despite this drop in wages, total labor income still goes up. The added income is used together with savings from lower consumption to invest in new capital-equipment, which is now more profitable. Once this new capital equipment is in place, the capital-skill ratio jumps up, as does the skilled wage.

Although the underlying wage dynamics associated with the two different types of shocks are quite dissimilar, their impact on the skill premium is quite similar. In Figure 8, we see that the ratio of unskilled to skilled hours worked initially rises in response to the investment-specific shock, while the capital-skill ratio falls. Together, they produce a skill premium which is uncorrelated with contemporaneous equipment-specific shocks. Thus, the skill premium is uncorrelated with contemporaneous shocks, regardless of the type of shock.

In both cases, the capital stock begins to grow in the period following the productivity shock. This leads to growth in the equipment-skill ratio, which in turn raises the skill premium. In fact, we can induce from the shape of the two impulse response functions of the model skill premium, that the capital-skill complementarity effect dominates the relative supply effect, creating a strong, positive lag in the skill premium in response to productivity shocks, regardless of the type of shock.

In summary, the benchmark model is able to explain 55 percent of the observed volatility in the skill premium. It produces a skill premium which is uncorrelated with contemporaneous output and which lags the business cycle. The behavior of both the equipment-skill ratio and relative hours worked matches the data quite well. On the other hand, it fails to explain 45 percent of the volatility in the skill premium, the point estimate of the contemporaneous correlation of the skill premium with output in the data lies below the confidence interval generated by the model simulations and the skill premium in the model peaks earlier than it does in the data. In the following section we shall see that these problems arise from a rather conservative benchmark calibration of the model and that they can be remedied by increasing the degree of capital-skill complementarity in the model.

4.3.1 Sensitivity Analysis

In this section, a sensitivity analysis is carried out which demonstrates that the results which speak in favor of the model with capital-skill complementarity are robust to significant parameter changes. It also shows that the model can be calibrated in such a way as to significantly improve its match with the data. The sensitivity analysis in this section also investigates a number of important questions which are related to the problem at hand. For example, it will examine how the skill premium behaves in a model without capital-skill complementarity and it will also highlight the role and importance of investment-specific technological shocks. The analysis concludes with a brief comment on the potential bias which may exist in the data on the skill premium used in this paper.

Table 5 summarizes the results (with respect to the skill premium) of the different experiments from this analysis. The information from the U.S. data is repeated in the first row of the table and the results from the benchmark calibration are also included.

The first exercise concerns the behavior of the skill premium in a model without capital-skill complementarity. Here, it is assumed that production is given by a 4-factor, Cobb-Douglas production function. Everything else in the model is left

Table 5: Sensitivity Analysis

x = Skill premium	Volatility of x	Cross-Correlations of output (t) and x x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)
Data	1.90	-.20	-.22	-.06	0.09	0.16
Benchmark calibration	1.04	0.10	0.12	0.17	0.15	0.11
	(0.15)	(0.12)	(0.13)	(0.13)	(0.13)	(0.15)
Model with...						
Cobb-Douglas production[a]	0.33	0.80	0.21	-.11	-.25	-.26
	(0.04)	(0.05)	(0.11)	(0.11)	(0.12)	(0.12)
$\rho_{zq} = 0^b$	1.09	0.25	0.39	0.20	0.08	0.00
	(0.14)	(0.11)	(0.11)	(0.13)	(0.14)	(0.14)
No investment-specific shocks[c]	0.30	0.00	0.75	0.69	0.48	0.26
	(0.04)	(0.07)	(0.02)	(0.06)	(0.10)	(0.12)
$\nu = 0.756^d$	1.90	0.03	0.13	0.19	0.17	0.13
	(0.28)	(0.13)	(0.13)	(0.12)	(0.13)	(0.13)

a) $\theta_{k_s} = 0.117$, $\theta_{k_e} = 0.183$, $\theta_s = 0.276$, $\theta_u = 0.424$

b) $\sigma_z = 0.0243$, c) $\sigma_z = 0.0255$, d) $\mu = 0.3503$

unchanged. In this case, the skill premium is a function of the relative hours worked only.

The volatility of the skill premium in this model is extremely low. It is approximately 17 percent of that found in the data. The skill premium is also strongly procyclical, which, of course, is not true for the data. In short, the model without capital-skill complementarity is not able to explain the cyclical behavior of the skill premium.

Next, the negative correlation between shocks to investment-specific and neutral technological change is removed. In Table 5, we see that this does not affect the volatility of the skill premium. It does however increase the magnitude of the correlations between output and the skill premium, which now peak at time $t + 1$.

Removing investment-specific shocks altogether, we find that the volatility of the skill premium drops from 1.04 (0.15) percent to a mere 0.30 (0.04) percent. Eliminating these shocks lowers the volatility of investment in capital equipment, which, in turn, affects the volatility of the equipment-skill ratio which falls from 4.50 (0.50) percent to 1.52 (0.50) percent. This, in turn, lowers the volatility of the skill premium, since the ratio of capital equipment to skilled hours worked is the main determinant of the skill premium in the model.

Removing investment-specific shocks also lowers the contemporaneous correlation between output and the skill premium to 0.00 (0.07). The skill premium peaks in period $t+1$ with a much stronger correlation than before. The correlation between time t output and the skill premium at time $t+1$ is 0.75 (0.07).

Thus, even without investment-specific shocks, the model with capital-skill complementarity produces a skill premium which is uncorrelated with contemporaneous output and that lags the business cycle. These two qualitative results are, in fact, very robust to parameter changes and even to alternative specifications of the production function.[16] But, as we saw above, investment-specific technological shocks are necessary for matching the volatility of the skill premium found in the data, as well as the magnitude and structure of the lagged correlations between output and the skill premium.

In general, investment-specific shocks tend to make the skill premium (weakly) procyclical. Their impact on output and the skill premium are more immediate than the effects produced by neutral technology shocks. Neutral shocks tend to make the skill premium acyclical or even (weakly) countercyclical. So, matching the data requires a balance between the two.

The elasticity of substitution between skilled and unskilled workers and between unskilled workers and capital, $1/(1-\nu)$, is obviously central to the analysis in this paper. The estimate of ν used in this paper was taken from Krusell et al. (2000) and was equal to 0.401, which implies an elasticity of substitution of 1.67. This estimate may, however, be somewhat on the conservative side, at least when compared to the existing literature.

If we examine the studies summarized in Hamermesh (1993), and restrict our-

[16] For example, we get the same result if we use a more simple, 3-factor CES production function which does not distinguish between equipment and structures.

selves to those which use aggregate or large industry data for the U.S., we find that the average elasticity of substitution between production workers and capital is equal to 1.96 (the highest reported estimate is equal to 2.92 and the lowest is equal to 0.91). The average elasticity of substitution between production and non-production workers is equal to 3 (the highest reported estimate is equal to 5.51 and the lowest is 0.49).[17]

Setting ν equal to 0.756, so that the elasticity of substitution between skilled and unskilled workers and between capital and unskilled workers is now equal to 4.1,[18] raises the volatility of the skill premium in the model to 1.90 (0.28) percent. It now matches the volatility of the skill premium in the data. The volatility of relative hours worked in the model increases to 1.91 (0.16) percent, making it a closer match to the 2.15 percent found in the data.

Raising the elasticity of substitution also lowers the contemporaneous correlation between the skill premium and output to 0.03 (0.13) and increases the value of the lags. Thus, strengthening the degree of capital-skill complementarity in the model improves the model's fit with the data. If we were to, instead, weaken the amount of capital-skill complementarity in the model, the model would perform more like the Cobb-Douglas model discussed above. Henceforth, this parameter calibration (with $\nu = 0.756$) will be referred to as the preferred model.

A final observation is that the contemporaneous correlation between output and the skill premium in the data is (weakly) negative. It is essentially zero in the preferred model. The correlation in the data falls below the confidence interval of the model estimate. This is true in all of the exercises above and in the initial, benchmark model. This result may be due to the presence of a composition and/or

[17] It is important to note, however, that not all of the elasticities in these studies are directly comparable to the substitution elasticities in this model. The substitution elasticities in the model are direct elasticities, while in Hamermesh's review, there are a good number of Allen partial elasticities as well.

[18] This is the direct elasticity between manual and nonmanual workers estimated by Dougherty (1972).

selection bias in the Krusell et al. (2000) data set. Controlling for these types of bias, Keane and Prasad (1993) found the skill premium to be uncorrelated with contemporaneous unemployment. Their finding matches the prediction of the model with capital-skill complementarity.[19]

5 A Final Test of the Model with Capital-Skill Complementarity

A final test of the model with capital-skill complementarity against the data can be made using the simulated time series for output and the skill premium generated by the preferred model in a VAR experiment. First, we construct a VAR(p) model of output, y_t, and the skill premium, $w_{s,t}/w_{u,t}$, using the Krusell et al. (2000) data. The preferred model specification is a VAR(2) without a constant[20]

$$\begin{bmatrix} y_t \\ \frac{w_{s,t}}{w_{u,t}} \end{bmatrix} = \mathbf{\Delta}_1 \begin{bmatrix} y_{t-1} \\ \frac{w_{s,t-1}}{w_{u,t-1}} \end{bmatrix} + \mathbf{\Delta}_2 \begin{bmatrix} y_{t-1} \\ \frac{w_{s,t-1}}{w_{u,t-1}} \end{bmatrix} + \boldsymbol{\varepsilon}_t \tag{17}$$

$$\boldsymbol{\varepsilon}_t \sim N(0, \mathbf{\Sigma})$$

where $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$ are a 2×2 matrix of OLS regression coefficients and where $\boldsymbol{\varepsilon}_t$ is a 2×1 vector of normally distributed *i.i.d.* shocks with mean zero and variance-covariance matrix $\mathbf{\Sigma}$. The impulse response function of the skill premium (in the data) to a one standard deviation shock to output (in the data) is shown in Figure 9.

The same experiment is then carried out using simulated output and the simu-

[19] Changes in the degree of risk aversion, the subjective discount rate and the depreciation rates of structures and equipment were also examined. They had no significant impact on the results and are, therefore, not reported here.

[20] This representation of the model is chosen since it minimizes both the Akaike information criteria and the Schwarz criteria.

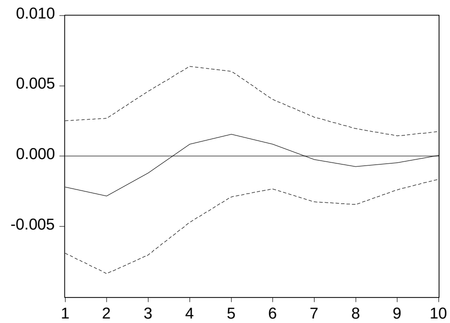

Figure 9: Impulse Response of the Skill Premium (from the data) to a One S.D. Shock to the Output (from the data) (along with $\pm$ 2 s.d. bands).

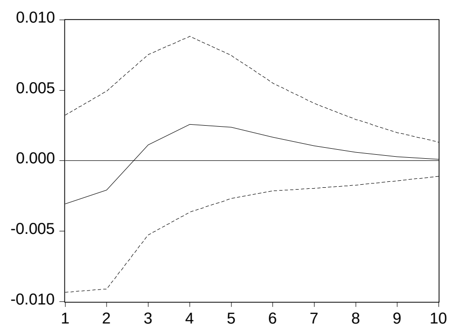

Figure 10: Impulse Response of the Skill Premium (from the preferred model) to a One S.D. Shock to the Output (from the preferred model) (along with $\pm$ 2 s.d. bands).

lated skill premium from the preferred model. The impulse response function from this experiment is shown in Figure 10. Examining these two figures we see that they resemble each other quite closely. Again, we see that the skill premium is essentially acyclical and that it lags the business cycle. This final test speaks quite strongly in favor of the model with capital-skill complementarity and its ability to explain movements in the skill premium over the business cycle.

6 Conclusion

The purpose of this study has been to investigate the implications of capital-skill complementarity for the cyclical behavior of wage inequality. The results of this study suggest that capital-skill complementarity may be an important determinant of wage inequality over the business cycle. The model with capital-skill complementarity can account for both the volatility and the cyclical behavior of the skill premium in the United States. The model without capital-skill complementarity cannot.

Previous research has shown that the capital-skill complementarity mechanism illustrated in this paper can also be used successfully to help us understand increasing trends in wage inequality (Krusell et al., 2000; Lindquist, 2000) and increasing wage dispersion (Caselli, 1999). Together, these and other studies, allow us to conclude that capital-skill complementarity is an important ingredient in a successful, competitive theory of relative wages and that such a theory can, in fact, help us to understand changes in the structure of relative wages.

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